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Analysis and Design for No-Spin Tethered Satellite Retrieval

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I. Introduction

TETHERED satellites are composed of two or more orbiting bodies connected by light, flexible members known as tethers. The study of tethered satellites began with Tsilokovsky in 1895 and was taken up by Artsutanov in the 1950s [1]. Since that time a number of researchers have studied aspects of the application of tethered satellite systems including the dynamics of tethered satellites and the architecture of tethered satellite missions [1–9]. Tethered satellites are of interest both because of the physical and mathematical problems they present and because of their many practical uses. Tethered satellite systems have been identified as candidates for novel atmospheric probes, interferometers, magnetometers, and gravity gradiometers among other devices [10–14]. Some far term applications of tethered satellites include the tethered artificial gravity (TAG) system and the momentum exchange and electrodynamic reboost (MXER) system. The TAG system is a device capable of imparting an artificial gravitational force to bodies on orbit [15], whereas the MXER system is a momentum exchange device designed to boost payloads into higher orbits without the use of chemical propellants [16]. Numerous other uses for tethered satellites in space are detailed in [17].

During the course of a tethered satellite mission, it may be necessary to shorten the overall length of the tether. The purpose of such a retrieval could be to facilitate the repair or servicing of the tethered satellite assembly, to reduce the profile of the orbital debris created by a satellite at the end of its useful life, or simply to alter the dynamical behavior of the tethered satellite system. We will show that in general, such a length contraction causes the tethered satellite system to enter a spin with respect to the orbital reference frame. In many of the instances described above, the reason for retrieving a tethered satellite system necessitates that the system not spin with respect to the orbital reference frame; such a maneuver could be achieved through the use of a sophisticated angular velocity control system. However, by exploiting the dynamics of a tethered satellite system, it is possible to reduce the length of the system without causing the system to enter a spin. Analysis of the equations of motion will reveal that under an exponential length control law, there exist initial tethered satellite states for which the tethered satellite system can be retrieved and remain stationary with respect to the orbital reference frame during the course of the retrieval maneuver. These points, which correspond to the equilibria of the system, will be identified and classified. We will describe the motion of a simplified tethered satellite system undergoing retrieval in the neighborhood of these equilibrium points.

II. Planar–Dumbbell Model

A. Model Definition and Equations of Motion

The analysis performed in this paper assumes that a two-bodied tethered satellite is composed of two point mass end-bodies that are connected by a massless, rigid tether. We also assume that out-of-plane motion is negligible. These assumptions together comprise the "planar–dumbbell" model of a two-bodied tethered satellite [15]. Note that although this model does not capture all of the factors affecting tethered satellite motion, the dynamics studied will propagate into more detailed tethered satellite models. Figure 1 gives a visual representation of the two-body model.

The equations of motion for a planar–dumbbell type satellite (e.g., as related by Mazzoleni and Hoffman [15]) are

$$\ddot{L} - L(\dot{\theta} + 2\Omega\dot{\theta} + 3\Omega^2\cos^2\theta) = -\frac{T_0}{\bar{m}} \tag{1}$$

and

$$\ddot{\theta} + 2\frac{\dot{L}}{L}(\Omega + \dot{\theta}) + \frac{3}{2}\Omega^2\sin 2\theta = 0 \tag{2}$$

where
1) L = distance between two end masses
2) θ = angle between tether and local vertical
3) T_0 = tension force transmitted by the tether
4) $\Omega = \sqrt{GM/r_0^3}$ = the orbital angular velocity
5) G = the universal graviational constant
6) M = the mass of the Earth
7) r_0 = the distance from the center of the Earth to the center of mass of the tethered satellite system
8) $\bar{m} = [m_a m_b/(m_a + m_b)]$ where m_a and m_b are the masses of the two end bodies

Equations (1) and (2) comprise a system of two differential equations in three unknown quantities (L, θ, and T_0). To close the system, we will assume throughout this paper that the tether length L follows the exponential control law

$$L(t) = L_i e^{-ct} \tag{3}$$

The dynamics associated with such an exponential control law have been the subject of many previous studies [18–22] and is detailed by Beletsky and Levin [23]. In Eq. (3), the quantity c is defined as

$$c = \frac{\ln(L_i/L_f)}{t_r} \tag{4}$$

where L_i is the initial tether length, L_f is the final tether length, and t_r is the time required for the reel-in procedure to complete. Note that

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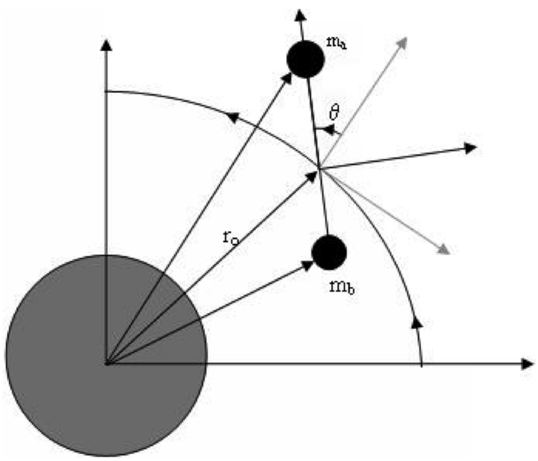

Fig. 1 A visual representation of the planar–dumbbell model.

Eqs. (2) and (3) can be used to completely describe the motion of the tethered satellite system with respect to the orbital reference frame.

B. Nondimensionalized and State Space Equations of Motion

To investigate the equations of motion Eqs. (1) and (2), we nondimensionalize them using the variable substitution suggested by Misra and Modi [4]

$$\tau = \Omega t \tag{5}$$

and

$$\tilde{L} = \frac{L}{L_i} \tag{6}$$

Thus

$$\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}\theta}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} = \Omega\theta'; \qquad \ddot{\theta} = \Omega^2\theta'' \tag{7}$$

where the superscripted prime indicates differentiation with respect to τ. Likewise

$$\frac{\mathrm{d}\tilde{L}}{\mathrm{d}\tau} = \Omega\tilde{L}'; \qquad \ddot{\tilde{L}} = \Omega^2\tilde{L}'' \tag{8}$$

Next define $\tilde{c} = (c/\Omega)$ so that

$$\frac{\dot{L}}{L} = -\tilde{c}\Omega \tag{9}$$

The nondimensional equations of motion are, therefore,

$$\theta'' - 2\tilde{c}(1 + \theta') + \tfrac{3}{2}\sin 2\theta = 0 \tag{10}$$

and

$$\tilde{L}'' - \tilde{L}(\theta'^2 + 2\theta' + 3\cos^2\theta) = -\frac{T_0}{\bar{m}\Omega^2 L_i} \tag{11}$$

Equation (10) can be cast into state form by making the substitutions

$$x_1 = \theta \tag{12}$$

$$x_2 = \theta' \tag{13}$$

Therefore the nondimensional state space equations of motion are

$$x_1' = x_2 \tag{14}$$

and

$$x_2' = 2\tilde{c}(1 + x_2) - \tfrac{3}{2}\sin 2x_1 \tag{15}$$

The equations of motion can be numerically integrated from different initial conditions as a function of the parameter $\tilde{c}$. The behavior of the solution trajectories in the phase plane has been explained in terms of the sets of phase-space points at which the equations of motion are exactly zero (i.e., the system nullclines) by Padgett and Mazzoleni [24].

III. Existence and Location of Equilibria

An equilibrium state of a system is any state in which the magnitude of the linear and angular momentum of a system is constant [25,26]; for the tethered satellite, the equilibria occur when

$$x_1' = x_2' = 0 \tag{16}$$

Solving Eqs. (14) and (15) simultaneously with Eq. (16) indicates that at equilibrium points we have

$$x_2 = 0 \tag{17}$$

and

$$x_1 = \tfrac{1}{2}\arcsin\tfrac{4\tilde{c}}{3} \tag{18}$$

The arcsin function in Eq. (18) is only defined if the magnitude of the argument is less than or equal to 1; that is, equilibria only exist if

$$-\tfrac{3}{4} < \tilde{c} < \tfrac{3}{4} \tag{19}$$

Such a result has been noted by many researchers (e.g., Kulla [27]). Because of the symmetry of the planar–dumbbell system, only two solutions of Eq. (18) represent unique configurations; furthermore these solutions are separated by no more than π radians. Without loss of generality, the x_1 interval of study for the purposes of this paper is therefore chosen to be $[-\pi/2, \pi/2]$.

Figure 2 shows the values of the equilibria that are present along the interval $x_1 = [-\pi/2, \pi/2]$ for $x_2 = 0$. For small values of $\tilde{c}$, Eq. (15) can be approximated by

$$x_2' = -\tfrac{3}{2}\sin 2x_1 \tag{20}$$

which, under the variable substitution $x_t = 2x_1$, yields

$$x_2' = -\tfrac{3}{2}\sin x_2 \tag{21}$$

which represents the state space equation of motion for a simple pendulum. Note that for small values of $\tilde{c}$, the equilibria positions plotted on Fig. 2 approach 0 and $\pi/2$ as is expected for a simple pendulum and that the equilibria disappear for values of $\tilde{c}$ greater than $\frac{3}{4}$; both of these observations reinforce previously discussed analytical traits of the system. In this paper, the equilibrium value closer to $(x_1, x_2) = (0, 0)$ will be known as the lower equilibrium and the value closer to $(x_1, x_2) = (\pi/2, 0)$ will be known as the upper equilibrium.

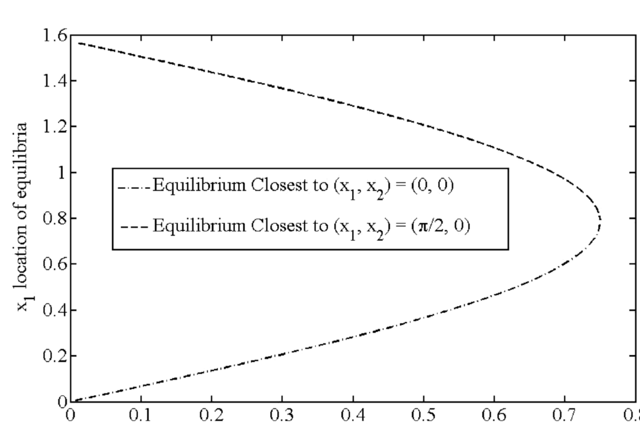

Fig. 2 Bifurcation diagram of the planar–dumbbell equations of motion.

IV. Classification of Equilibria

To classify the equilibria of the tethered satellite system, we examine the eigenvalues of the linear approximation of the equations of motion in the neighborhood of each equilibrium. The eigenvalues of the linear approximation can give clues as to the qualitative nature of the phase–plane behavior of the equations of motion [24,25]. The linear approximation about a point $(x_1, 0)$ is given by

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3\cos 2x_1 & 2\tilde{c} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{22}$$

The eigenvalues of the 2×2 matrix in Eq. (22) are given by

$$\lambda_{1,2} = \tilde{c} \pm \sqrt{\tilde{c}^2 - 3\cos 2x_1} \tag{23}$$

Equation (23) gives values of the eigenvalues in the neighborhood of the equilibria resulting from any given value of $\tilde{c}$. Figure 3 shows the values taken by $\lambda_{1,2}$. Equation (23) shows that if the discriminant quantity is less than zero, the eigenvalues of the system will have nonzero imaginary parts. Figure 3 shows that for most of the values of $\tilde{c}$ for which equilibria exist, the discriminant of the lower equilibrium (the equilibrium that is closest to the origin of the phase plane) is less than zero and thus the eigenvalues corresponding to the lower equilibrium have nonzero imaginary parts. Furthermore Fig. 3 shows that the discriminant of the eigenvalues corresponding to the upper equilibrium is greater than zero for all values of $\tilde{c}$ for which equilibria exist.

The characteristic phase–plane solution trajectory of a linear system with imaginary eigenvalues is a spiral around the equilibrium at which the eigenvalues of the linearization of the system are imaginary and a trajectory proceeding either toward the equilibrium (known as a sink), away from the equilibrium (known as a source), or some combination of the two (known as a saddle) [25]. Solution trajectories in the neighborhood of the lower equilibrium are likely to have a spiral structure around the equilibrium whereas solution trajectories in the neighborhood of the upper equilibrium will exhibit behavior characteristic of a node. Therefore let the lower equilibrium be known as the spiral equilibrium and the upper equilibrium be known as the nodal equilibrium. The assertion concerning solution trajectory behavior in the neighborhood of an equilibrium is confirmed by numerically simulating solution trajectories in the phase plane. This simulation is shown again in Fig. 4. The spiral structure around the spiral equilibrium and the much different solution structure around the nodal equilibrium can clearly be seen in Fig. 4.

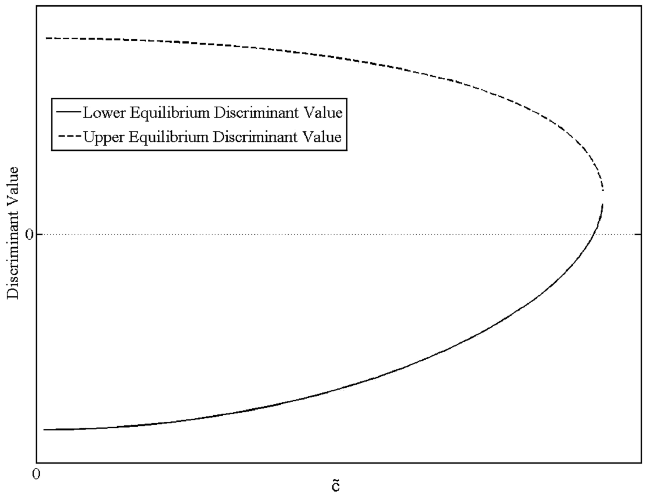

Fig. 3 The value of the discriminant of the eigenvalues. Positive values indicate real eigenvalues whereas negative values indicate imaginary eigenvalues.

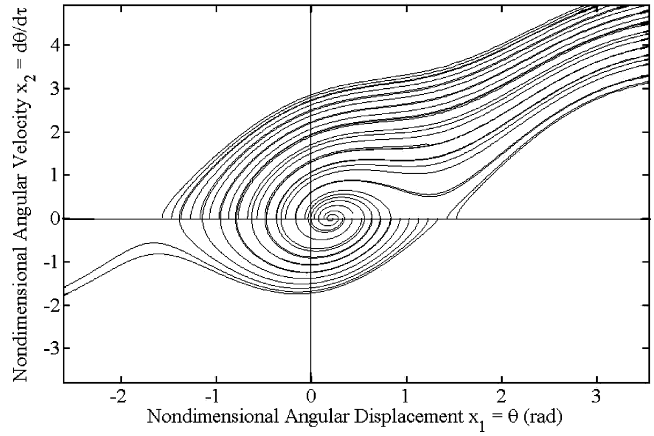

Fig. 4 The behavior of some sample solution trajectories in the phase plane.

V. Solution Behavior in the Neighborhood of the Lower Equilibrium

For a tether with initial conditions near the spiral equilibrium point, the resulting solution trajectories proceed slowly around the spiral equilibrium. If the solution trajectory of a tethered satellite does not travel far from the equilibrium before the reel-in process is complete, the system will oscillate rather than spin with respect to the orbital reference frame. This type of behavior is shown in Fig. 5. Each of the solution trajectories begin near the spiral equilibrium. When the reel-in procedure terminates, the solution trajectories stop moving out and away from the spiral equilibrium and instead move in orbits around the new system equilibrium at $(x_1, x_2) = (0, 0)$, indicating oscillating motion. Each of the orbits corresponds to pendular oscillations.

To investigate the nature of the solution trajectories, we simulate the system for times greater than t_r; the tethered satellite length decreases on the interval $t = [0, t_r]$ and the length remains constant at points in time greater than t_r. At the end of such a simulation, the tethered satellite is either spinning or oscillating. Figure 6 is colored black for the $\theta_0 - \tilde{c}$ pairs that result in an oscillating system and white for the pairs that result in a spinning system. In performing the calculations necessary to create Fig. 6, oscillating systems were detected by observing the changes in the sign of the angular velocity of the system occurring after the reel-in process terminates. After the end of the reel-in procedure, the tethered satellite system behaves like a simple pendulum. If the system angular velocity changes sign, the system is in an oscillating state; if the sign of the system angular velocity remains constant, the system is in a spinning state. Figure 6 also shows lines corresponding to the location in the $\theta_0 - \tilde{c}$ plane of both the lower equilibrium (white line) and the upper equilibrium (dashed, gray line). Note that the area of the plot surrounding the lower equilibrium is dominated by oscillating satellite states.

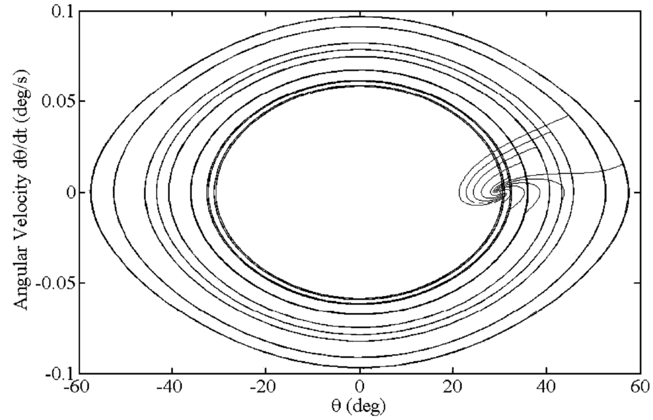

Fig. 5 The value of the discriminant of the eigenvalues. Positive values indicate real eigenvalues whereas negative values indicate imaginary eigenvalues.

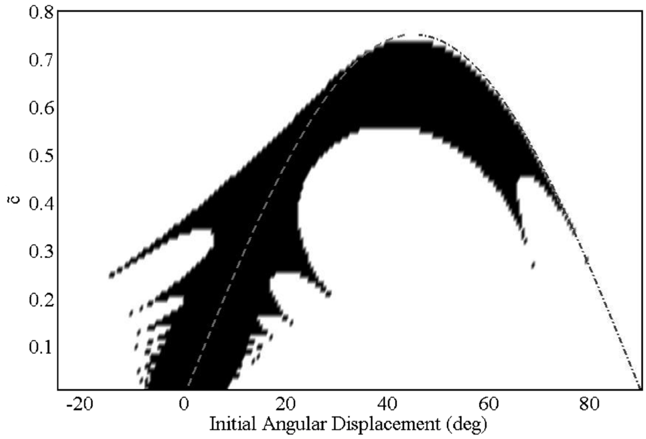

Fig. 6 θ_0–$\tilde{c}$ pairs that result in oscillating systems (black) and spinning systems (white).

Figure 6 further reinforces the assertion that solution trajectories in the neighborhood of the lower equilibrium evolve relatively slowly and tend to circulate about the lower equilibrium point, indicating that when the reel-in procedure ends, the tethered satellite systems corresponding to such solution trajectories will oscillate rather than spin.

VI. Solution Behavior in the Neighborhood of the Upper Equilibrium

The physical behavior of a tethered satellite system in the neighborhood of the upper solution trajectory can be inferred from Fig. 4. If the initial angular displacement of a tethered satellite system is greater than the value of the upper equilibrium (but still within the $[-\pi/2, \pi/2]$ range) the solution trajectory of the tethered satellite system begins to the right of the upper equilibrium and the tethered satellite system enters a spin in the positive θ (counterclockwise) direction. Furthermore if the initial angular displacement of a tethered satellite system is less than the θ value of the upper equilibrium (but greater than the θ value of the lower equilibrium) the solution trajectory proceeds downward and to the right in the phase plane. The solution trajectory may or may not circulate around the equilibrium. The net effect is that solution trajectories tend to be repelled from the upper equilibrium whereas solution trajectories tend to circulate about the lower equilibrium; this is consistent with the analytical classification of the upper equilibrium as a node.

VII. Summary

We have examined the dynamics of a tethered satellite system undergoing a length contraction maneuver. In general, a tethered satellite system being reeled in will spin with respect to the orbital reference frame. We have shown that under simplified tethered satellite models, it is possible for a length reduction maneuver to be performed without causing the tethered satellite system to spin. We have shown that such a maneuver requires the initial conditions of the length reduction maneuver to be located in the neighborhood of the spiral equilibrium. The numerical simulations that we have presented indicate that length reductions performed in the neighborhood of the spiral equilibrium will result in systems that are stationary with respect to the orbital reference frame during the length reduction maneuver and that will oscillate with respect to the orbital reference frame after the length reduction maneuver is complete.

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